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SQUARING THE CIRCLE

(*Journal of the Indian Mathematical Society*, v, 1913, 132)

Let PQR be a circle with centre O, of which a diameter is PR. Bisect PO at H and let T be the point of trisection of OR nearer R. Draw TQ perpendicular to PR and place the chord $RS = TQ$.

Join PS, and draw OM and TN parallel to RS. Place a chord $PK = PM$, and draw the tangent $PL = MN$. Join RL, RK and KL. Cut off $RC = RH$. Draw CD parallel to KL, meeting RL at D.

Then the square on RD will be equal to the circle PQR approximately.

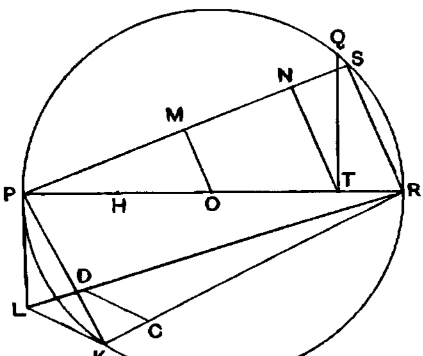
For
$$RS^2 = \tfrac{5}{36} d^2,$$
where d is the diameter of the circle.

Therefore
$$PS^2 = \tfrac{31}{36} d^2.$$

But PL and PK are equal to MN and PM respectively.

Therefore
$$PK^2 = \tfrac{31}{144} d^2, \text{ and } PL^2 = \tfrac{31}{324} d^2.$$

Hence
$$RK^2 = PR^2 - PK^2 = \tfrac{113}{144} d^2,$$
and
$$RL^2 = PR^2 + PL^2 = \tfrac{355}{324} d^2.$$



But
$$\frac{RK}{RL} = \frac{RC}{RD} = \frac{3}{2}\sqrt{\frac{113}{355}},$$
and
$$RC = \tfrac{3}{4} d.$$

Therefore
$$RD = \frac{d}{2}\sqrt{\frac{355}{113}} = r\sqrt{\pi}, \text{ very nearly.}$$

Note.—If the area of the circle be 140,000 square miles, then RD is greater than the true length by about an inch.